Equal tempered scale
Calculation of note frequency

The equal tempered scale, also called "the equal temperament".

There are a number of different scale and/or temperament systems: the Pythagorean system, the Zarlino scale, the mesotonic temperaments, the unequal temperaments, the tempered scale, etc.
Unlike other systems, the tempered scale is characterised by the division of the octave into twelve perfectly equal semitones. This simplification is necessary in order to tune, once and for all, instruments with fixed tones, i.e., those whose pitch cannot be adjusted by the player, like the violinist, during performance. It is a compromise that allows the instrument to play in all keys and allows all transpositions. As soon as a formation includes one or more fixed-sounding instruments, the use of equal temperament is mandatory for all except for historical reconstructions. The tempered scale is the standard in Western music.

Reference frequency

The frequency of the A_4 is fixed at 440 Hz.

Octaves

The ratio of the frequencies of two notes that differ by an octave is equal to 2.
"*Going up an octave*" is equivalent to "*multiplying the frequency by 2*". For example frequency $A_4 = 440$ Hz; frequency $A_5 = 880$ Hz; frequency $A_6 = 1760$ Hz; etc.
"*Going down one octave*" is equivalent to "*dividing the frequency by 2*":
frequency $A_3 = 220$ Hz; frequency $A_2 = 110$ Hz.
Note that sound intervals perceived as equal correspond to frequencies in geometric progression. It is said that sound perception is logarithmic.

Half-tones

The equal tempered range is characterised by equal semitones. Let r be the ratio of the frequencies of two consecutive semitones:
"*going up a semitone*" is equivalent to "*multiplying the frequency by r*".
Since the octave is divided into 12 equal semitones, it can be said that "*going up an octave*" is equivalent to "*going up 12 semitones*", which leads to the equation

$$2 = r^{12}$$

The ratio of the frequencies of two successive semitones can be deduced from this:

$$r = \sqrt[12]{2} \simeq 1.05946$$

Thus, the difference of a comma between $C_\sharp$ and $D_\flat$ is erased, allowing these two notes to be associated with the same key on a keyboard.

Frequency of a musical note

According to the rule "*going up a semitone*" is equivalent to "*multiplying the frequency by r*", the frequency of the notes can be calculated:
frequency $(A\sharp)_4 = (440 \text{ Hz}) \times r \simeq 466.16$ Hz
frequency $(B)_4 = (440 \text{ Hz}) \times (r^2) \simeq 493.88$ Hz, etc.

According to the rule "*going down half a tone*" is equivalent to "*dividing the frequency by r.*", the frequency of the notes can be calculated:

frequency $(G\sharp)_4 = \frac{440 \text{ Hz}}{r} \simeq 415.3$ Hz

frequency $(G)_4 = \frac{440 \text{ Hz}}{r^2} \simeq 392$ Hz, etc.

The formula is therefore

$$\text{frequency(note)} = (440 \text{ Hz})\, r^n$$

where n is the number of semitones between the A_4 and the note, counted positively upwards or negatively downwards, for example

$$\text{frequency}\,(C_5) = (440 \text{ Hz})\, r^3 \simeq 523.251 \text{ Hz}$$

$$\text{frequency}\,(D_4) = (440 \text{ Hz})\, r^{-7} \simeq 293.665 \text{ Hz}$$

The equal temperate intervals

The equal tempered fifth, made up of seven semitones, corresponds to the frequency ratio

$$r^7 \simeq 1.498307$$

instead of $\frac{3}{2} = 1.5$ for the pure fifth. This slight shortening makes the wolf's fifth disappear.

Frequency table in hertz

	1	2	3	4	5	6	7	8	9	10
C	32.703	65.406	130.81	261.63	523.25	1046.5	2093.	4186.	8372.	16744.
C♯=D♭	34.648	69.296	138.59	277.18	554.37	1108.7	2217.5	4434.9	8869.8	17740.
D	36.708	73.416	146.83	293.66	587.33	1174.7	2349.3	4698.6	9397.3	18795.
D♯=E♭	38.891	77.782	155.56	311.13	622.25	1244.5	2489.	4978.	9956.1	19912.
E	41.203	82.407	164.81	329.63	659.26	1318.5	2637.	5274.	10548.	21096.
F	43.654	87.307	174.61	349.23	698.46	1396.9	2793.8	5587.7	11175.	22351.
F♯=G♭	46.249	92.499	185.	369.99	739.99	1480.	2960.	5919.9	11840.	23680.
G	48.999	97.999	196.	392.	783.99	1568.	3136.	6271.9	12544.	25088.
G♯=A♭	51.913	103.83	207.65	415.3	830.61	1661.2	3322.4	6644.9	13290.	26580.
A	55.	110.	220.	440.	880.	1760.	3520.	7040.	14080.	28160.
A♯=B♭	58.27	116.54	233.08	466.16	932.33	1864.7	3729.3	7458.6	14917.	29834.
B	61.735	123.47	246.94	493.88	987.77	1975.5	3951.1	7902.1	15804.	31609.

Tuning of instruments with fixed sounds (keyboard instruments and guitars)

To facilitate the tuning of an instrument, the smartphone can be turned into a frequency meter by downloading an application. In the case of a *spectral analyser*, the frequency to be taken into account is the smallest frequency, called the fundamental, the others being the harmonics.

Hypertext link to the father page:

Music on the website www.deleze.name
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