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Edition 2017

Thème : Calcul d' erreur
Lien vers les énoncés des exercices :
https://www.deleze.name/marcel/sec2/applmaths/csud/calcul_erreur/1_a_2-calcul_erreur.pdf

Corrigé de l'exercice 1 - 1 [sans ordinateur]

Calculons d'abord la valeur

$$R = A\cos(\varphi) = 0.3\cos(27\,°) = 0.2673$$

Calculons ensuite les dérivées partielles

$$\frac{\partial R}{\partial A} = \frac{\partial}{\partial A}(A\cos(\varphi)) = \cos(\varphi)\frac{\partial}{\partial A}(A) = \cos(\varphi)$$

$$\frac{\partial R}{\partial \varphi} = \frac{\partial}{\partial \varphi}(A\cos(\varphi)) = A\frac{\partial}{\partial \varphi}(\cos(\varphi)) = A(-\sin(\varphi)) = -A\sin(\varphi)$$

Substituons dans la formule de Gauss-Laplace

$$\Delta A = 0.02\,A = 0.02 * 0.3 = 0.006$$

$$\Delta\varphi = 1\,° = \frac{\pi}{180} \simeq 0.0175 \qquad [\text{radians}]$$

$$\Delta R = \sqrt{(\cos(\varphi)\,\Delta A)^2 + (-A\sin(\varphi)\,\Delta\varphi)^2} =$$
$$\sqrt{(\cos(27\,°)\,0.006)^2 + (-0.3\sin(27\,°)\,0.0175)^2} \approx 0.00585$$

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$R \approx 0.267 \pm 0.006$$

Corrigé de l'exercice 1 - 1 [avec *Mathematica*]

Calculons d'abord la valeur

```
valeurs = {A → 0.3, φ → 27 °};

erreurs = {ΔA → 0.02 A, Δφ → 1 °};

R = A Cos[φ] /. valeurs
      cosinus
0.267302
```

$$\sqrt{(\partial_A(A\,\mathrm{Cos}[\varphi])\,\Delta A)^2 + (\partial_\varphi(A\,\mathrm{Cos}[\varphi])\,\Delta\varphi)^2}$$

$$\sqrt{\Delta A^2\,\mathrm{Cos}[\varphi]^2 + A^2\,\Delta\varphi^2\,\mathrm{Sin}[\varphi]^2}$$

$$\Delta R = \sqrt{(\partial_A(A\,\mathrm{Cos}[\varphi])\,\Delta A)^2 + (\partial_\varphi(A\,\mathrm{Cos}[\varphi])\,\Delta\varphi)^2}\ \texttt{/. erreurs /. valeurs}$$

```
0.0058507
```

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$R \approx 0.267 \pm 0.006$$

Corrigé de l'exercice 1 - 2 a) [sans ordinateur]

Calculons les dérivées partielles

$$\frac{\partial \rho}{\partial r} = \frac{\partial}{\partial r}\left(\frac{3\,m}{4\,\pi\,r^3}\right) = \frac{3\,m}{4\,\pi}\,\frac{\partial}{\partial r}\left(r^{-3}\right) = \frac{3\,m}{4\,\pi}\left(-3\,r^{-4}\right) = \frac{-9\,m}{4\,\pi\,r^4}$$

$$\frac{\partial \rho}{\partial m} = \frac{\partial}{\partial m}\left(\frac{3\,m}{4\,\pi\,r^3}\right) = \frac{3}{4\,\pi\,r^3}\,\frac{\partial}{\partial m}(m) = \frac{3}{4\,\pi\,r^3}$$

Substituons dans la formule de Gauss-Laplace

$$\Delta\rho = \sqrt{\left(\frac{-9\,m}{4\,\pi\,r^4}\,\Delta r\right)^2 + \left(\frac{3}{4\,\pi\,r^3}\,\Delta m\right)^2}$$

Calculons l'erreur relative sur ρ en fonction des erreurs relatives sur r et m :

$$\frac{\Delta\rho}{\rho} = \sqrt{\frac{\left(\frac{-9\,m}{4\,\pi\,r^4}\,\Delta r\right)^2 + \left(\frac{3}{4\,\pi\,r^3}\,\Delta m\right)^2}{\left(\frac{3\,m}{4\,\pi\,r^3}\right)^2}} = \sqrt{\left(\frac{-9\,m}{4\,\pi\,r^4}\,\frac{4\,\pi\,r^3}{3\,m}\,\Delta r\right)^2 + \left(\frac{3}{4\,\pi\,r^3}\,\frac{4\,\pi\,r^3}{3\,m}\,\Delta m\right)^2} =$$

$$\sqrt{\left(\frac{-3}{r}\,\Delta r\right)^2 + \left(\frac{1}{m}\,\Delta m\right)^2} = \sqrt{9\left(\frac{\Delta r}{r}\right)^2 + \left(\frac{\Delta m}{m}\right)^2} = \sqrt{9\,(0.02)^2 + (0.005)^2} \approx 0.0602$$

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$\frac{\Delta\rho}{\rho} \approx 6\,\%$$

Corrigé de l'exercice 1 - 2 b) [Calcul numérique avec *Mathematica*]

```
erreurs = {Δr → 0.02 r, Δm → 0.005 m};
```

Erreur absolue

$$\sqrt{\left(\partial_r\left(\frac{m}{\frac{4}{3}\,\pi\,r^3}\right)\Delta r\right)^2 + \left(\partial_m\left(\frac{m}{\frac{4}{3}\,\pi\,r^3}\right)\Delta m\right)^2}$$

$$\sqrt{\frac{9\,\Delta m^2}{16\,\pi^2\,r^6} + \frac{81\,m^2\,\Delta r^2}{16\,\pi^2\,r^8}}$$

Erreur relative

$$\sqrt{\frac{\left(\partial_r\left(\frac{m}{\frac{4}{3}\,\pi\,r^3}\right)\Delta r\right)^2 + \left(\partial_m\left(\frac{m}{\frac{4}{3}\,\pi\,r^3}\right)\Delta m\right)^2}{\left(\frac{m}{\frac{4}{3}\,\pi\,r^3}\right)^2}} \;/.\; \text{erreurs}$$

0.060208

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$\frac{\Delta\rho}{\rho} \approx 6\,\%$$

Corrigé de l'exercice 1 - 2 c) [Calcul littéral avec *Mathematica*]

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\frac{3\,m}{4\,\pi\,r^3}$$

Erreur absolue

$$\Delta\rho = \text{Simplify}\left[\sqrt{\left(\partial_r\left(\frac{m}{\frac{4}{3}\pi r^3}\right)\Delta r\right)^2 + \left(\partial_m\left(\frac{m}{\frac{4}{3}\pi r^3}\right)\Delta m\right)^2}, \{m > 0, r > 0\}\right]$$

simplifie

$$\frac{3\sqrt{r^2\,\Delta m^2 + 9\,m^2\,\Delta r^2}}{4\,\pi\,r^4}$$

Erreur relative

$$i\rho = \text{Simplify}\left[\frac{\Delta\rho}{\rho}, \{m > 0, r > 0\}\right]$$

simplifie

$$\frac{\sqrt{r^2\,\Delta m^2 + 9\,m^2\,\Delta r^2}}{m\,r}$$

Corrigé de l'exercice 1 - 3 a) [sans ordinateur]

Calculons d'abord les valeurs z_1, z_2, z_3

$$z_1 = \frac{1.34 * 4.34^2}{0.027} \simeq 934.804$$

$$z_2 = \frac{1.36 * 4.35^2}{0.025} \simeq 1029.38$$

$$z_3 = \frac{1.35 * 4.34^2}{0.026} \simeq 978.002$$

Calculons les erreurs Δz_1, Δz_2, Δz_3

$$\frac{\partial}{\partial a}\left(\frac{a\,b^2}{c}\right) = \frac{b^2}{c}\,\frac{\partial}{\partial a}(a) = \frac{b^2}{c}$$

$$\frac{\partial}{\partial b}\left(\frac{a\,b^2}{c}\right) = \frac{a}{c}\,\frac{\partial}{\partial b}\left(b^2\right) = \frac{a}{c}\,2\,b = \frac{2\,a\,b}{c}$$

$$\frac{\partial}{\partial c}\left(\frac{a\,b^2}{c}\right) = a\,b^2\,\frac{\partial}{\partial c}\left(c^{-1}\right) = a\,b^2\left(-c^{-2}\right) = \frac{-a\,b^2}{c^2}$$

$$\Delta\left(\frac{a\,b^2}{c}\right) = \sqrt{\left(\frac{b^2}{c}\Delta a\right)^2 + \left(\frac{2\,a\,b}{c}\Delta b\right)^2 + \left(\frac{-a\,b^2}{c^2}\Delta c\right)^2}$$

$$\Delta z_1 = \sqrt{\left(\left(\frac{4.34^2}{0.027}\,0.03\right)^2 + \left(\frac{2 * 1.34 * 4.34}{0.027}\,0.02\right)^2 + \left(\frac{-1.34 * 4.34^2}{0.027^2}\,0.004\right)^2\right)} \simeq 140.3$$

$$\Delta z_2 = \sqrt{\left(\left(\frac{4.35^2}{0.025}\,0.03\right)^2 + \left(\frac{2 * 1.36 * 4.35}{0.025}\,0.02\right)^2 + \left(\frac{-1.36 * 4.35^2}{0.025^2}\,0.004\right)^2\right)} \simeq 166.5$$

$$\Delta z_3 = \sqrt{\left(\left(\frac{4.34^2}{0.026}\,0.03\right)^2 + \left(\frac{2 * 1.35 * 4.34}{0.026}\,0.02\right)^2 + \left(\frac{-1.35 * 4.34^2}{0.026^2}\,0.004\right)^2\right)} \simeq 152.3$$

Calculons la moyenne

$$z = \frac{z_1 + z_2 + z_3}{3} = \frac{1}{3}\,(934.804 + 1029.38 + 978.002) \simeq 980.729$$

Calculons enfin l'erreur sur la moyenne

$$\frac{\partial}{\partial z_1}\left(\frac{z_1 + z_2 + z_3}{3}\right) = \frac{1}{3}\,\frac{\partial}{\partial z_1}(z_1 + z_2 + z_3) = \frac{1}{3}$$

$$\frac{\partial}{\partial z_2}\left(\frac{z_1 + z_2 + z_3}{3}\right) = \frac{1}{3}\,\frac{\partial}{\partial z_2}(z_1 + z_2 + z_3) = \frac{1}{3}$$

$$\frac{\partial}{\partial z_3}\left(\frac{z_1 + z_2 + z_3}{3}\right) = \frac{1}{3}\,\frac{\partial}{\partial z_3}(z_1 + z_2 + z_3) = \frac{1}{3}$$

$$\Delta z = \sqrt{\left(\frac{1}{3}\,\Delta z_1\right)^2 + \left(\frac{1}{3}\,\Delta z_2\right)^2 + \left(\frac{1}{3}\,\Delta z_3\right)^2} = \frac{1}{3}\sqrt{(140.3)^2 + (166.5)^2 + (152.3)^2} \simeq 88.57$$

Finalement

$$z \simeq 981 \pm 89 \approx 980 \pm 90$$

Corrigé de l'exercice 1 - 3 b) [avec *Mathematica*]

Calculons d'abord les valeurs z_1, z_2, z_3

```
valeurs1 = {{a → 1.34, b → 4.34, c → 0.027},
   {a → 1.36, b → 4.35, c → 0.025}, {a → 1.35, b → 4.34, c → 0.026}};
```

```
erreurs1 = {Δa → 0.03, Δb → 0.02, Δc → 0.004};
```

$$\frac{a\,b^2}{c} \;/.\; \text{valeurs1}$$

```
{934.804, 1029.38, 978.002}
```

Calculons les erreurs Δz_1, Δz_2, Δz_3

$$\sqrt{\left(\partial_a\left(\frac{a\,b^2}{c}\right)\Delta a\right)^2 + \left(\partial_b\left(\frac{a\,b^2}{c}\right)\Delta b\right)^2 + \left(\partial_c\left(\frac{a\,b^2}{c}\right)\Delta c\right)^2} \;/.\; \text{erreurs1} \;/.\; \text{valeurs1}$$

```
{140.327, 166.529, 152.29}
```

Calculons la moyenne

```
valeurs2 = {z1 → 934.8, z2 → 1029.4, z3 → 978.0};
```

```
erreurs2 = {Δz1 → 140.3, Δz2 → 166.5, Δz3 → 152.3};
```

$$z = \frac{z1 + z2 + z3}{3} \;/.\; \text{valeurs2}$$

```
980.733
```

Calculons enfin l'erreur sur la moyenne

$$\Delta z = \sqrt{\left(\left(\partial_{z1}\left(\frac{z1+z2+z3}{3}\right)\Delta z1\right)^2 + \left(\partial_{z2}\left(\frac{z1+z2+z3}{3}\right)\Delta z2\right)^2 + \left(\partial_{z3}\left(\frac{z1+z2+z3}{3}\right)\Delta z3\right)^2\right)} \;/.$$

```
   erreurs2 /. valeurs2
88.5699
```

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$z \simeq 981 \pm 89 \approx 980 \pm 90$$

Corrigé de l'exercice 2 - 1

Pour une fonction linéaire $z = f(x) = a\,x$, on a

$$I(a\,x) = \sqrt{\left(\frac{\partial}{\partial x}(a\,x)\right)^2 \Delta x^2} = \sqrt{(a)^2\,\Delta x^2} = |a|\,\Delta x$$

$$i(a\,x) = \frac{I(a\,x)}{|a\,x|} = \frac{|a|\,\Delta x}{|a|\;|x|} = \frac{\Delta x}{|x|} = i(x)$$

Fonction affine

$$I(a\,x + b) = \sqrt{\left(\frac{\partial}{\partial x}(a\,x+b)\right)^2 \Delta x^2} = \sqrt{(a)^2\,\Delta x^2} = |a|\,\Delta x = |a|\;I(x)$$

Fonctions puissances

$$I\left(x^2\right) = \sqrt{\left(\frac{\partial}{\partial x}\left(x^2\right)\right)^2 \Delta x^2} = \sqrt{(2\,x)^2\,\Delta x^2} = 2\sqrt{x^2}\;\Delta x$$

$$i\left(x^2\right) = \frac{I\left(x^2\right)}{|x^2|} = \frac{2\sqrt{x^2}\;\Delta x}{x^2} = 2\sqrt{\frac{x^2}{x^4}}\;\Delta x = 2\sqrt{\frac{1}{x^2}}\;\Delta x = 2\,\frac{\Delta x}{|x|} = 2\,i(x)$$

$$I\left(\sqrt{x}\right) = \sqrt{\left(\frac{\partial}{\partial x}\left(x^{\frac{1}{2}}\right)\right)^2 \Delta x^2} = \sqrt{\left(\frac{1}{2}\,x^{-\frac{1}{2}}\right)^2 \Delta x^2} = \frac{1}{2}\sqrt{\frac{1}{x}}\;\Delta x$$

$$i\left(\sqrt{x}\right) = \frac{\frac{1}{2}\sqrt{\frac{1}{x}}\;\Delta x}{|\sqrt{x}|} = \frac{1}{2}\sqrt{\frac{\frac{1}{x}}{x}}\;\Delta x = \frac{1}{2}\sqrt{\frac{1}{x^2}}\;\Delta x = \frac{1}{2}\,\frac{\Delta x}{|x|} = \frac{1}{2}\,i(x)$$

$$I\left(\frac{1}{x^3}\right) = \sqrt{\left(\frac{\partial}{\partial x}\left(x^{-3}\right)\right)^2 \Delta x^2} = \sqrt{\left(-3\,x^{-4}\right)^2 \Delta x^2} = 3\sqrt{\frac{1}{x^8}}\;\Delta x$$

$$i\left(\frac{1}{x^3}\right) = \frac{3\sqrt{\frac{1}{x^8}}\;\Delta x}{\left|\left(\frac{1}{x^3}\right)\right|} = 3\sqrt{\frac{x^6}{x^8}}\;\Delta x = 3\sqrt{\frac{1}{x^2}}\;\Delta x = 3\,\frac{\Delta x}{|x|} = 3\,i(x)$$

$$I\left(x^n\right) = \sqrt{\left(\frac{\partial}{\partial x}\left(x^n\right)\right)^2 \Delta x^2} = \sqrt{\left(n\,x^{n-1}\right)^2 \Delta x^2} = |n|\sqrt{\left(x^{n-1}\right)^2}\;\Delta x$$

$$i\left(x^n\right) = \frac{|n| \sqrt{\left(x^{n-1}\right)^2}\, \Delta x}{\left(|x\right)^n|} = |n|$$

$$\sqrt{\left(\frac{x^{n-1}}{x^n}\right)^2}\, \Delta x = |n| \sqrt{\left(\frac{1}{x}\right)^2}\, \Delta x = |n| \frac{\Delta x}{|x|} = |n|\, i(x)$$

Fonctions trigonométriques

$$I(\cos(x)) =$$

$$\sqrt{\left(\frac{\partial}{\partial x}(\cos(x))\right)^2 \Delta x^2} = \sqrt{(-\sin(x))^2 \Delta x^2} = |\sin(x)|\, \Delta x = |\sin(x)|\, I(x)$$

$$I(\sin(x)) =$$

$$\sqrt{\left(\frac{\partial}{\partial x}(\sin(x))\right)^2 \Delta x^2} = \sqrt{(\cos(x))^2 \Delta x^2} = |\cos(x)|\, \Delta x = |\cos(x)|\, I(x)$$

$$I(\tan(x)) = \sqrt{\left(\frac{\partial}{\partial x}(\tan(x))\right)^2 \Delta x^2} = \sqrt{\left(\frac{1}{\cos^2(x)}\right)^2 \Delta x^2} = \frac{1}{\cos^2(x)} \Delta x = \frac{1}{\cos^2(x)} I(x)$$

Somme et différence de variables indépendantes

$$I(x+y) = \sqrt{\left(\frac{\partial}{\partial x}(x+y)\right)^2 \Delta x^2 + \left(\frac{\partial}{\partial y}(x+y)\right)^2 \Delta y^2} =$$

$$\sqrt{1^2 \Delta x^2 + 1^2 \Delta y^2} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{I^2(x) + I^2(y)}$$

$$I(x-y) = \sqrt{\left(\frac{\partial}{\partial x}(x-y)\right)^2 \Delta x^2 + \left(\frac{\partial}{\partial y}(x-y)\right)^2 \Delta y^2} =$$

$$\sqrt{1^2 \Delta x^2 + (-1)^2 \Delta y^2} = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{I^2(x) + I^2(y)}$$

Quotient de variables indépendantes

$$I\left(\frac{x}{y}\right) =$$

$$\sqrt{\left(\frac{\partial}{\partial x}\left(\frac{x}{y}\right)\right)^2 \Delta x^2 + \left(\frac{\partial}{\partial y}\left(\frac{x}{y}\right)\right)^2 \Delta y^2} = \sqrt{\left(\frac{1}{y}\right)^2 \Delta x^2 + \left(-\frac{x}{y^2}\right)^2 \Delta y^2} = \sqrt{\left(\frac{1}{y}\right)^2 \Delta x^2 + \left(\frac{x}{y^2}\right)^2 \Delta y^2}$$

$$i\left(\frac{x}{y}\right) = \frac{\sqrt{\left(\frac{1}{y}\right)^2 \Delta x^2 + \left(\frac{x}{y^2}\right)^2 \Delta y^2}}{\frac{x}{y}} = \sqrt{\frac{y^2}{x^2}\left(\left(\frac{1}{y}\right)^2 \Delta x^2 + \left(\frac{x}{y^2}\right)^2 \Delta y^2\right)} =$$

$$\sqrt{\left(\frac{1}{x}\right)^2 \Delta x^2 + \left(\frac{1}{y}\right)^2 \Delta y^2} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} = \sqrt{i^2(x) + i^2(y)}$$

Corrigé de l'exercice 2 - 2

Calculons d'abord la valeur

$$R = A\cos(\varphi) = 0.3\cos(27\,°) = 0.2673$$

Utilisons successivement

la formule de propagation des incertitudes relatives sur le produit

la formule de propagation des incertitudes absolues sur le cosinus

$$\frac{\Delta R}{|R|} = i(R) =$$

$$\sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta(\cos(\varphi))}{\cos(\varphi)}\right)^2} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{-\sin(\varphi)\,\Delta\varphi}{\cos(\varphi)}\right)^2} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + (\tan(\varphi)\,\Delta\varphi)^2}$$

dans laquelle on substitue les valeurs numériques

$$\frac{\Delta A}{A} = 0.02 \qquad \Delta\varphi = 1\,° = \frac{\pi}{180} \simeq 0.0175$$

$$\frac{\Delta R}{|R|} = i(R) = \sqrt{(0.02)^2 + (\tan(27\,°)\ 0.0175)^2} \simeq 0.0219$$

L'incertitude absolue sur *R* vaut

$$\Delta R = i(R)\,|R| = 0.0219 * 0.2673 = 0.00585$$

La réponse est arrondie à un ou deux chiffres caractéristique(s) :

$$R = 0.2673 \pm 0.0059 \approx 0.267 \pm 0.006$$

Corrigé de l'exercice 2 - 3

En utilisant les règles de calcul du § 2, on a

$$i\left(\frac{m}{\frac{4}{3}\pi r^3}\right) = i\left(\frac{m}{r^3}\right) = \sqrt{(i(m))^2 + \left(i\left(r^{-3}\right)\right)^2} =$$

$$\sqrt{i^2(m) + (3\,i(r))^2} = \sqrt{i^2(m) + 9\,i^2(r)} \simeq \sqrt{0.005^2 + 9 * 0.02^2} \simeq 0.060208 \simeq 6\,\%$$

Corrigé de l'exercice 2 - 4

Calculons d'abord les valeurs z_1, z_2, z_3

$$z_1 = \frac{1.34 * 4.34^2}{0.027} \simeq 934.804$$

$$z_2 = \frac{1.36 * 4.35^2}{0.025} \simeq 1029.38$$

$$z_3 = \frac{1.35 * 4.34^2}{0.026} \simeq 978.002$$

Calculons les incertitudes relatives puis absolues

$$i\left(\frac{a\,b^2}{c}\right) = \sqrt{(i(a))^2 + \left(i\left(b^2\right)\right)^2 + (i(c))^2} =$$

$$\sqrt{(i(a))^2 + (2\,i(b))^2 + (i(c))^2} = \sqrt{(i(a))^2 + 4\,(i(b))^2 + (i(c))^2}$$

$$\frac{\Delta z_1}{z_1} = i(z_1) \simeq \sqrt{\left(\frac{0.03}{1.34}\right)^2 + 4\left(\frac{0.02}{4.34}\right)^2 + \left(\frac{0.004}{0.027}\right)^2} \simeq 0.150113$$

$$\triangle z_1 = I(z_1) \approx 0.150113 * 934.804 \simeq 140.327$$

$$\frac{\triangle z_2}{z_2} = i(z_2) \simeq \sqrt{\left(\frac{0.03}{1.36}\right)^2 + 4\left(\frac{0.02}{4.35}\right)^2 + \left(\frac{0.004}{0.025}\right)^2} \simeq 0.161775$$

$$\triangle z_2 = I(z_2) \approx 0.161775 * 1029.38 \simeq 166.528$$

$$\frac{\triangle z_3}{z_3} = i(z_3) \simeq \sqrt{\left(\frac{0.03}{1.35}\right)^2 + 4\left(\frac{0.02}{4.34}\right)^2 + \left(\frac{0.004}{0.026}\right)^2} \simeq 0.155716$$

$$\triangle z_3 = I(z_3) \approx 0.155716 * 978.002 \simeq 152.29$$

Calculons enfin la moyenne et l'erreur sur la moyenne

$$z = \frac{z_1 + z_2 + z_3}{3} = \frac{1}{3}(934.804 + 1029.38 + 978.002) \simeq 980.729$$

$$I\left(\frac{z_1 + z_2 + z_3}{3}\right) = \frac{1}{3} I(z_1 + z_2 + z_3) =$$

$$\frac{1}{3}\sqrt{(I(z_1))^2 + (I(z_2))^2 + (I(z_3))^2} = \frac{1}{3}\sqrt{(140.3)^2 + (166.5)^2 + (152.3)^2} \simeq 88.57$$

Finalement

$$z = 981 \pm 89 = 980 \pm 90$$